Gravitational-Wave (Astro)Physics: from Theory to Data and Back

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Spitzer Lectures

• **Lecture I**: Basics of gravitational-wave theory and modeling

• **Lecture II**: Advanced methods to solve the two-body problem in General Relativity

• **Lecture III**: Inferring cosmology and astrophysics with gravitational-wave observations

• **Lecture IV**: Probing dynamical gravity and extreme matter with gravitational-wave observations

(NR simulation: Ossokine, AB & SXS @AEI)

(visualization credit: Benger @ Airborne Hydro Mapping Software & Haas @AEI)
Given current tight constraints on GR (e.g., Solar system, binary pulsars), can any GR deviation be observed with GW detectors?
PN templates in stationary phase approximation: TaylorF2

\[ \tilde{h}(f) = A_{SPA}(f) e^{i\psi_{SPA}(f)} \]

\[ \psi_{SPA}(f) = 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128} (\pi M f)^{-5/3} \{1 + \]

\[ - \frac{5\lambda^2}{336\omega_{BD}} \nu^{2/5} (\pi M f)^{-2/3} - \frac{128}{3} \frac{\pi^2 D M}{\lambda_g^2 (1 + z)} (\pi M f)^{2/3} \]

\[ + \left( \frac{3715}{756} + \frac{55}{9} \nu \right) \nu^{-2/5} (\pi M f)^{2/3} - 16\pi \nu^{-3/5} (\pi M f) + 4\beta \nu^{-3/5} (\pi M f) \]

\[ + \left( \frac{15293365}{508032} + \frac{27145}{504} \nu + \frac{3085}{72} \nu^2 \right) \nu^{-4/5} (\pi M f)^{4/3} - 10\sigma \nu^{-4/5} (\pi M f)^{4/3} \} \]

\[ \beta = \frac{1}{12} \sum_{i=1}^{2} \chi_i \left[ 113 \frac{m_i^2}{M^2} + 75\nu \right] \widehat{L} \cdot \widehat{S}_i, \quad \sigma = \frac{\nu}{48} \chi_1 \chi_2 \left( -27 \widehat{S}_1 \cdot \widehat{S}_2 + 721 \widehat{L} \cdot \widehat{S}_1 \widehat{L} \cdot \widehat{S}_2 \right) \]

\[ \chi_i = \frac{S_i}{m_i^2} \]

graviton with non zero mass

spin-orbit

1.5PN

1.5PN

2PN

2PN

spin-spin
Bounding PN parameters: inspiral

- GW150914/GW122615’s rapidly varying orbital periods allow us to bound higher-order PN coefficients in gravitational phase.

\[
\tilde{h}(f) = A(f)e^{i\varphi(f)}
\]
\[
\varphi(f) = \varphi_{\text{ref}} + 2\pi f t_{\text{ref}} + \varphi_{\text{Newt}}(Mf)^{-5/3}
\]
\[
+ \varphi_{0.5\text{PN}}(Mf)^{-4/3} + \varphi_{1\text{PN}}(Mf)^{-3/3}
\]
\[
+ \varphi_{1.5\text{PN}}(Mf)^{-2/3} + \ldots
\]

(Abbott et al. PRX6 (2016))

(Arun et al. 06, Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)

- PN parameters describe: tails of radiation due to backscattering, spin-orbit and spin-spin couplings.

- PN parameters take different values in modified theories to GR.
Some modified theories to General Relativity

<table>
<thead>
<tr>
<th>Theory</th>
<th>(\alpha_{ppE})</th>
<th>(a_{ppE})</th>
<th>(\beta_{ppE})</th>
<th>(b_{ppE})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jordan–Fierz–Brans–Dicke</td>
<td>(-\frac{5}{96}\frac{S^2}{\omega_{BD}}\eta^{2/5})</td>
<td>-2</td>
<td>(-\frac{5}{3584}\frac{S^2}{\omega_{BD}}\eta^{2/5})</td>
<td>-7</td>
</tr>
<tr>
<td>Dissipative Einstein-Dilaton-Gauss–Bonnet gravity</td>
<td>0</td>
<td>.</td>
<td>(-\frac{5}{7168}\zeta_3\eta^{-18/5}\delta_m^2)</td>
<td>-7</td>
</tr>
<tr>
<td>Massive Graviton</td>
<td>0</td>
<td>.</td>
<td>(-\frac{\pi^2 DM_c}{\lambda^2_\gamma(1+z)})</td>
<td>-3</td>
</tr>
<tr>
<td>Lorentz Violation</td>
<td>0</td>
<td>.</td>
<td>(-\frac{\pi^2-\gamma_{LV}}{(1-\gamma_{LV})}\frac{D_{\gamma_{LV}}}{\lambda^2_{\gamma_{LV}}(1+z)^{-1-\gamma_{LV}}})</td>
<td>(-3\gamma_{LV} - 3)</td>
</tr>
<tr>
<td>(G(t)) Theory</td>
<td>(-\frac{5}{512}\dot{G}M_c)</td>
<td>-8</td>
<td>(-\frac{25}{65536}\dot{G}_cM_c)</td>
<td>-13</td>
</tr>
<tr>
<td>Extra Dimensions</td>
<td>0</td>
<td>.</td>
<td>(-\frac{75}{2554344}\frac{dM}{dt}\eta^{-4}(3-26\eta+24\eta^2))</td>
<td>-13</td>
</tr>
<tr>
<td>Non-Dynamical Chern–Simons Gravity</td>
<td>(\alpha_{PV})</td>
<td>3</td>
<td>(\beta_{PV})</td>
<td>6</td>
</tr>
<tr>
<td>Dynamical Chern–Simons Gravity</td>
<td>0</td>
<td>.</td>
<td>(\beta_{dCS})</td>
<td>-1</td>
</tr>
</tbody>
</table>

(Yunes & Siemens 2013)
Bounding phenom parameters: intermediate/merger-RD

\[ \varphi(f) = \varphi_{\text{ref}} + 2\pi f t_{\text{ref}} + \varphi_{\text{Newt}}(M f)^{-5/3} \]
\[ + \varphi_{0.5\text{PN}}(M f)^{-4/3} + \varphi_{1\text{PN}}(M f)^{-1} \]
\[ + \varphi_{1.5\text{PN}}(M f)^{-2/3} + \cdots + \beta_2 \log(M f) \]
\[ + \cdots + \alpha_4 \tan^{-1}(a M f + b) \]

- Merger-ringdown phenomenological parameters \((\beta_i \text{ and } \alpha_i)\) not yet expressed in terms of relevant parameters in GR and modified theories of GR.

(Abbott et al. PRL 116 (2016) 221101)
Tests of Lorentz Invariance/Bounding Graviton Mass

- Phenomenological approach: modified dispersion relation. GWs travel at speed different from speed of light. (Will 94, Mirshekari, Yunes & Will 12)

\[ E^2 = p^2 c^2 + Ap^\alpha c^\alpha \]
\[ \alpha \geq 0 \]
\[ \frac{v_g}{c} = 1 + (\alpha - 1)\frac{A}{2}E^{\alpha-2} \]

\[ m_g \leq 7.7 \times 10^{-23} \text{eV}/c^2 \]
\[ \alpha = 0, \ A > 0 \]
\[ \lambda_g = \frac{\hbar}{m_g c} \]

(Abbott et al. PRL118 (2017))
How to test GR and probe nature of compact objects: building deviations from GR & BHs/NSs

• Do current GR waveform models include all physical effects? Not yet.

• Will GR deviations be fully captured in perturbative-like descriptions during merger-ringdown stage? Likely not. (e.g., Yunes & Pretorius 09, Li et al. 12, Endlich et al. 17)

• Need NRAR waveforms in modified theories of GR: scalar-tensor theories, Einstein-Aether theory, dynamical Chern-Simons, Einstein-dilaton Gauss-Bonnet theory, massive gravity theories, etc. (e.g., Stein et al. 17, Cayuso et al. 17, Hirschmann et al. 17)

• Need NRAR waveforms of binaries composed of exotic objects (BH & NS mimickers), such as boson stars, gravastar, etc. (e.g., Palenzuela et al. 17)

• Including deviations from GR in EOB formalism. (Julie & Deruelle 17, Julie 17, Khalil et al. in prep 18)
Probing nature of remnant: quasi-normal modes (QNMs)

• **Deformed/perturbed black holes** emits quasi-normal modes.
• **Measuring** at least two modes will be *smoking gun* that Nature’s black holes are black holes of **General Relativity**.

- **Multiple QNMs** can be measured with future detectors, thus testing **no-hair conjecture** and **second-law black-hole mechanics** \((\text{Israel 69, Carter 71; Hawking 71, Bardeen 73})\).
Measuring BH’s mass and spin from multiple QNMs

\[ \Omega_{n\ell m} = M\omega_{n\ell m} = \left( 2\pi F_{n\ell m} + \frac{i}{T_{n\ell m}} \right) \text{Im}(\Omega) \]

- By knowing only one frequency and decay time, we cannot identify final BH’s mass and spin.

- Which SNRs are needed to measure multiple modes?
Black-hole spectroscopy by making full use of GW modeling

- **BH spectroscopy: unveiling nature of merger’s remnant**

  (Brito, AB & Raymond 18)

- We employ **parametrized inspiral-merger-ringdown** waveform model (pEOBNR) that includes modes beyond the dominant (2,2).

  • Using pEOBNR we recover **more stringent bounds** on frequency and decay time of GW150914 QNM, than using damped sinusoid model.

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**Graphical Representation**

- **GW150914**:
  - Mass ratio = 6
  - **PEOBNR** compared to different modes (e.g., $|h_{22}|$, $3 \times |h_{33}|$, etc.)

- **Frequency-mass ratio graph**
  - $f_{220}$ (Hz) vs. $\tau_{220}$ (ms)
  - Dashed and solid curves represent different models and assumptions (e.g., 1ms, 3ms, 5ms).

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**Equation and Formulas**

- Mass ratio $= 6$
- Frequency $f_{220}$
- Decay time $\tau_{220}$
Black-hole spectroscopy by making full use of GW modeling

(Brito, AB & Raymond 18)

• Let us assume we **did not find deviations from GR.**
• We bound **quasi-normal mode frequencies & decay times** by combining several BH observations. \( \sigma_{lm} = \sigma_{lm}^{GR} (1 + \delta \sigma_{lm}) \)

![Graph](image1.png)

**one event GW150914-like with Advanced LIGO & Virgo**

![Graph](image2.png)

**GW150914-like events in Advanced LIGO & Virgo**

• About **30 GW150914-like events** are needed to achieve errors of **5%** and test no-hair conjecture.
Remnant: black hole or exotic compact object (ECO)?

- If remnant is horizonless, and/or horizon is replaced by “surface”, new modes in the spectrum, and ringdown signal is modified: echoes signals emitted after merger.

(Damour & Solodukhin 07, Cardoso, Franzin & Pani 16)

horizonless objects
wormhole
black hole

boson stars, fermion stars, etc. (e.g., Giudice et al. 16)

(Cardoso et al. 16)
Constraints on speed of GWs & test of equivalence principle

- **Combining GW and GRB observations:**
  \[
  \frac{\Delta c}{c} \simeq c \frac{\Delta t}{D} \quad \text{assuming GRB is emitted 10 s after GW signal}
  \]
  \[
  \frac{\Delta c}{c} \leq 7 \times 10^{-16} \quad \text{assuming observed time delay is entirely due to different speed}
  \]
  \[
  \Delta t = t_{\text{EM}} - t_{\text{GW}}
  \]
  \[
  \Delta c = c_{\text{GW}} - c
  \]
  \[
  -4 \times 10^{-15} \leq \frac{\Delta c}{c} \leq 7 \times 10^{-16}
  \]

- **Strong constraints on scalar-tensor and vector-tensor theories of gravity.**
  (Creminelli et al. 17, Ezquiaga et al. 17, Sakstein et al. 17, Baker et al. 17)

- **EM waves & GWs follow same geodesic. Metric perturbations** (e.g., due to potential between source and Earth) **affect their propagation in same way.**
  \[
  \delta t_s = -\frac{1 + \gamma}{c^3} \int_{r_c}^{r_0} U(r(l)) dl
  \]
  \[
  -2.6 \times 10^{-7} \leq \gamma_{\text{GW}} - \gamma_{\text{EM}} \leq 1.2 \times 10^{-6}
  \]
  \[
  (Shapiro 1964)
  \]
  \[
  \text{gravitational potential of Milky Way outside sphere of 100 kpc}
  \]
  \[
  (Abbott et al. APJ 848 (2017) L12)
  \]
Solving two-body problem in General Relativity (including radiation)

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- **GR** is non-linear theory. Complexity similar to QCD.

- Einstein’s field equations can be solved:
  - approximately, but analytically (fast way)
  - exactly, but numerically on supercomputers (slow way)

- **Synergy** between analytical and numerical relativity is crucial.

- **GW170817**: SNR=32 (strong), 3000 cycles (from 30 Hz), one minute.

  last 0.07 sec modeled by AR
  last minutes modeled by NR

(Abbott et al. PRL 119 (2017) 161101)
Analytical waveform modeling for GW170817

- PN waveform model was used for:
  - **template bank**: to observe GW170817
  - **Bayesian analyses**: to infer astrophysical, fundamental physics information of GW170817
Probing equation of state of neutron stars

Neutron Star:
- mass: 1-3 Msun
- radius: 9-15 km
- core density > $10^{14}$ g/cm$^3$

- NS equation of state (EOS) affects gravitational waveform during late inspiral, merger and post-merger.

(tidal interactions)

(credit: Hinderer)
Probing equation of state of neutron stars

- Tidal effects imprinted on gravitational waveform during inspiral through parameter $\lambda$.

- $\lambda$ measures star’s quadrupole deformation in response to companion perturbing tidal field:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$
PN templates in stationary phase approximation: TaylorF2

\[ \tilde{h}(f) = A_{SPA}(f) \ e^{i \psi_{SPA}(f)} \]

\[ \psi_{SPA}(f) = 2\pi ft_c - \Phi_c - \frac{\pi}{4} + \frac{3}{128} \left( \pi M f \right)^{-5/3} \left\{ 1 + \frac{5\lambda^2}{336\omega_{BD}} \nu^{2/5} \left( \pi M f \right)^{-2/3} - \frac{128}{3} \frac{\pi^2 D M}{\lambda_g^2 (1 + z)} \left( \pi M f \right)^{2/3} \right\} \]

\[ + \left( \frac{3715}{756} + \frac{55}{9} \nu \right) \nu^{-2/5} \left( \pi M f \right)^{2/3} \left( \pi M f \right)^{-2/5} \nu^{-3/5} \left( \pi M f \right) + 4\beta \nu^{-3/5} \left( \pi M f \right) \]

\[ + \left( \frac{15293365}{508032} + \frac{27145}{504} \nu + \frac{3085}{72} \nu^2 \right) \]

\[ \cdots - \frac{39}{2} \nu^{-2} \tilde{\Lambda} \left( \pi M f \right)^{10/3} \}

\[ \tilde{\Lambda} = \frac{16 \left( m_1 + 12m_2 \right) m_1^4 \Lambda_1 + \left( m_2 + 12m_1 \right) m_2^4 \Lambda_2}{13 \left( m_1 + m_2 \right)^5} \]

\[ \lambda = \frac{M_{NS}}{m_{NS}^5} = \frac{2}{3} k_2 \left( \frac{R_{NS} c^2}{G m_{NS}} \right)^5 \]

- Dipole radiation
- Tidal
- Spin-orbit
- Spin-spin
- Graviton with non-zero mass
- Depends on EOS & compactness
- It can be large
Probing equation of state of neutron stars

- Where in frequency the information about (intrinsic) binary parameters predominantly comes from.  
  
  *(Harry & Hinderer 17)*

- Tidal effects typically change overall number of GW cycles from 30 Hz (about 3000) by one single cycle!
State-of-art waveform models for binary neutron stars

• Synergy between analytical and numerical work is crucial.
Tides make gravitational interaction more attractive
Constraining Love numbers with GW170817

(Abbott et al. PRL 119 (2017) 161101)

\[ \Lambda = \frac{\lambda}{m_{\text{NS}}^5} = \frac{2}{3} k_2 \left( \frac{R_{\text{NS}}c^2}{Gm_{\text{NS}}} \right)^5 \]

- Effective tidal deformability enters GW phase at 5PN order:

\[ \tilde{\Lambda} = \frac{16 (m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{13 (m_1 + m_2)^5} \]

- With state-of-art waveform models, tides are reduced by \(~20\%\). More analyses are ongoing.

Depends on EOS & compactness

\(|x| \leq 0.05\)

black hole

NS's Love number

more compact

less compact

APR4

SLy

H4

MPA1

MS1

MS1b

more compact

less compact

\Lambda_1

\Lambda_2

\Lambda_2
Boson stars as black-hole/neutron-star mimickers

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi} - \nabla^\alpha \Phi \nabla_\alpha \Phi^* - V(|\Phi|^2) \right] \]

<table>
<thead>
<tr>
<th>Type</th>
<th>Effective Potential</th>
<th>Max. Mass</th>
<th>Compactness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mini BS</td>
<td>(\mu^2\Phi^2)</td>
<td>(\frac{85\text{peV}}{\mu}) (M_\odot)</td>
<td>0.08</td>
</tr>
<tr>
<td>Massive BS</td>
<td>(\mu^2\Phi^2 + \frac{\lambda}{2}</td>
<td>\Phi</td>
<td>^4)</td>
</tr>
<tr>
<td>Neutron star</td>
<td>(\mu^2\Phi^2 \left(1 - \frac{2</td>
<td>\Phi</td>
<td>^2}{\sigma_0^2}\right)^2)</td>
</tr>
<tr>
<td>Solitonic BS</td>
<td>(\mu^2\Phi^2 \left(1 - \frac{2</td>
<td>\Phi</td>
<td>^2}{\sigma_0^2}\right)^2)</td>
</tr>
<tr>
<td>Black hole</td>
<td></td>
<td>(\infty)</td>
<td></td>
</tr>
</tbody>
</table>

\(C = \frac{GM}{Rc^2}\)

\((\text{Sennett...AB et al. 17})\)
\((\text{see also Cardoso et al. 17})\)

\(\Lambda = \lambda / M^5\)

- **Black holes:** \(\Lambda = 0\)
- **Neutron stars:** \(\Lambda_{\text{min}} \approx 10\)
- **Boson stars:** \(\Lambda_{\text{min}} \approx 1\)

\(\sigma_0 = 0.05 m_P\)
\(\mu = 10^{-10} \text{ eV}\)

\(\text{Stable} - \text{Unstable}\)

Boson star
The new era of precision gravitational-wave astrophysics

- Theoretical groundwork in **analytical and numerical relativity** has allowed us to build **faithful waveform models** to **search** for signals, **infer properties** and **test GR**.

- We can now **learn about gravity** in the genuinely **highly dynamical, strong field regime**.

- We can probe **matter under extreme pressure** and **density**.

- We have new ways to **explore relationships** between **gravity, light, particles and matter**.

- As for any new observational tool, gravitational (astro)physics will likely **unveil phenomena and objects never imagined** before.
“Astrophysical & Cosmological Relativity” Department

• Current members

• Past members contributed to work presented